Computer Implementation of Bankruptcy Rules

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Abstract:

In this paper we describe a computer system that calculates allocation to claims according to two kinds of bankruptcy rules. The first kind implemented are those whose results corresponds to the classical solutions of game theory: The Talmud rule, the Proportional rule, the Adjusted Proportional rule, the Constrained Equal Awards rule and the Random Arrival rule. The second type implemented generates assignations to claims that have no correspondence with the classical results, they are: Piniles´ rule, the Constrained Egalitarian rule and Constrained Equal Losses rule.

This computer system was developed in order to compute the different allocations to claims in Allocation problems, distribution problems and Assignment problems for real cases and to be used in education of game theory and decision problems. The system generates numerical and graphical results. It allows comparisons between the numerical results and their graphic representations. It also allows the exportation of the different outputs to any windows application.

Keywords: Computer System – Bankruptcy Rules - Cooperative Games Education – Game Theory Teaching.

Temas de Interés del Congreso: Optimización y Simulación - Informática Educativa.

1.- Introduction

When a firm goes bankrupt an important question to answer is which is the best way of splitting its remaining capital among its creditors.

A pioneer work in this matter was done by O'Neal (1982). He analyzed several assignation rules used from ancestral time and studied its properties. A good survey of the most common rules used in the literature of the bankruptcy problems is given by Thomson, W. (2003).

In this paper we present a computer implementation to the most classical rules corresponding to Game Theory solutions for solving bankruptcy problems and other traditional rules.

The computer system for bankruptcy problems is a software that computes the way of sharing a value among n agents. The system is developed for a Windows environment, using Delphi language. The system was mainly developed for being used in teaching and research on Cooperative Game Theory. It can be also used for analyzing bargaining problems and cost allocations problems. Lawyers and accountants can use it when a firm goes bankruptcy (according to the Argentinean Law 24.522).

This article extends the results given in Saavedra et al. (2003), by implementing new solutions: Piniles´ rule, the Constrained egalitarian rule and Constrained Equal losses rule. The system presents new tools and is entirely reprogrammed in Delphi 7.0.

The solutions are shown in graphical and analytical way. They can be exported to other applications.

2.- Bankruptcy Rules.

We will introduce the class of problems to be studied. The capital E of a firm has to be divided among a group of n claimants, $\mathbf{c_i}$ is the claim of agent $\mathbf{i} \in N$ (where N is a subset of the natural numbers and n the cardinality of N) and $c = (c_i)_{i \in N}$ is the claims vector. The sum of the claims is greater than the firm capital (the worth of the estate).

Definition 1: A bankruptcy problem is a pair $(c,E) \in R_+^n \times R_+$ such that $\sum ci \geq E$. β^N denotes the class of this problems. [1]

[1] We denotes with R_+^n the Cartesian product of n replies of R_+ indexed by 1,....n. By convenience we include the equality in the sum $\sum ci = E$.

Definition 2 : A bankruptcy rule, or just a rule, is a function that associates each bankruptcy problem $(c,E) \in \beta^N$ with a vector $\chi \in R_+^n$ which coordinates sum up E : $\sum \chi i = E$. We will use several instances of this vector denoted by $T_i(c,E)$, $P(c, E)$, $P^t_i(c, E)$, etc.

2.1 Rules with results corresponding to the classical Solutions[2]

2.1.1 Proportional Rule, P

The most common assignment rule is the proportional: the assignations are proportional to the claims. The proportionality is usually a good criteria and has been used from ancient time.

Definition 3: For all $(c, E) \in \beta^N$, $P(c, E) = \lambda c$, where λ is chosen so that $\sum \lambda c_i = E$ ($\lambda = E / \sum c_i$ if $c \neq 0$) .

2.1.2 Truncated-claims Proportional Rule, P^t

A version of the proportional rule is obtained by making awards proportional to the claims truncated by the worth of the estate:

Definition 4: For all $(c, E) \in \beta^N$ and all $i \in N$, $P^t_i(c, E) = \lambda \min\{c_i, E\}$, where λ is chosen so that $\sum \lambda \min\{c_i, E\} = E$ ($\lambda = E / \sum \min \{c_i , E\}$ if $c \neq 0$) .

The difference with the previous rule is that this rule does not consider those claims greater than the worth of the estate. Those claims that are greater than the worth of the estate are truncated to the worth of the estate.

2.1.3 Adjusted Proportional Rule, P^a

This rule first requires calculating for each agent an amount that can be interpreted as his "minimal right".

Definition 5 : For all $(c, E) \in \beta^N$ and all $i \in N$, Let $m_i(c,E) = \max\{E - \sum_{j \in N \setminus \{i\}} c_j, 0\}$ be the minimal right of claimant i ; also, let $m(c, E) = (m_i(c, E))_{i \in N}$. This is the amount that remains if every other agent receives his claim or zero if that remain is negative. Each claimant receives his minimal right and the remaining is divided proportionally among all of them.

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[2] Nash Bargaining Solution, Weighted Nash Bargaining Solution, Kalai-Smorodinsky bargaining solution, extended equal losses bargaining solutions. Coalitional Games: Shapley Value, Nucleolus, Dutta-Ray Solution and T-Value (see Thomson, W. (2003)).

Definition 6: For all $(c, E) \in \beta^N$ and all $i \in N$, $A_i(c,E) = m_i(c, E) + P(c - m(c, E), E - \sum m_i(c, E))$.[3]

2.1.4 Constrained Equal Award Rule, CEA

This rule awards each claimant the same amount regarding not to give to any of them more than his claim.

Definition 7: For all $(c, E) \in \beta^N$ and all $i \in N$, $CEA_i(c, E) = \min\{c_i, \lambda\}$, where λ is chosen so that $\sum \min\{c_i, \lambda\} = E$.[4]

The system divides the worth of the estate by the number of claimants (initial λ), then assigns each of them his claim if it is less than λ else assigns λ. No one receives more than he claims and the remaining between the claim and λ in the cases that λ is greater than the claim, is reallocated among those who claims more than λ.

2.1.5 Talmud Rule, T

From ancient time many discussions and recommendations have been done following other criteria different from the proportionality. An important and classical rule is given in the Talmud[5].

This rule, considers two different cases: 1) when the worth of the state is less or equal to the half of the sum of the claims and 2) when it is greater.

Definition 8: For all $(c,E) \in \beta^N$ and all $i \in N$, **1)** If $\sum c_i/2 \geq E$ then $T_i(c,E) = \min\{c_i/2, \lambda\}$, where λ is chosen so that $\sum\min\{c_i/2,\lambda\}= E$. **2)** If $\sum c_i/2 \leq E$ then $T_i(c,E) = c_i - \min\{c_i/2, \lambda\}$, where λ is chosen so that $\sum [c_i - \min\{ c_i/2, \lambda \}] = E$.

The algorithm we implemented works as follows for each case. This implementation is based on the proposal of Aumann & Maschler (1985).

1) We consider the worth of the estate to increase from 0 to half the sum of the claims: the first units are divided equally until each claimant has received an amount equal to half of the smallest claim. Then the claimant with the smallest claim stops receiving anything; instead, any additional unit is divided equally among all others until each of them has received an amount equal to half of the second smallest claim. Then

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[3] Curiel, Maschler y Tijs, 1987.
[4] Dutta, B. and D. Ray, 1989.
[5] Civil and religious code of the Jews.

the claimant with the second smallest claim stops receiving anything too and so on until no more money remains.

2) For estates worth more than $\sum c_i/2$ awards are computed in a symmetric way. First, each claimant receives his claim and this claim is reduced considering shortfalls of increasing size: initial shortfalls are divided equally until all claimants experience a loss equal to half of the smallest claim, at that point, the smallest claimant stops loosing. The others go on loosing until their common loss is equal to half of the second smallest claim. The process continues until the estate is worth $\sum c_i/2$.

2.1.6 Random Arrival Rule, RA

Now we consider claimants arriving one at a time to get their claims and we suppose that each claim is fully paid until money runs out. The result of the awards will depend on the order in which claimants arrive. To obtain independence, we take the arithmetic average over all orders of arrivals . This proposal was made by O'Neill (1982) and implemented by our computer system.

Definition 9: For all $(c, E) \in \beta^N$ and all $i \in N$, $RA_i(c, E) = 1/n! \sum_{\pi \in \Pi} N \min\{c_i, \max\{E - \sum_{j \in N, \pi(j) < \pi(i)} c_j, 0\}\}$. (where Π^n is the set of permutations of N).

2.2 Rules that provide results different from the classical ones.

2.2.1. Piniles´ Rule[6].

This rule awards each claimant in the same way CEA does but using the half of his claim instead of the whole claim, in the case that amount available is less or equal to the half of the sum of the claims. Otherwise, each claimant receives the half of his claim plus the amount resulting of the application of the CEA rule, still using the half of his claim and the remaining money.

Definition 10: For all $(c, E) \in \beta^N$ and all $i \in N$, $Pin_i(c, E) = CEA_i(c/2, E)$ if $\sum (c_j/2) >= E$ and $Pin_i(c, E) = c_i/2 + CEA_i(c/2, E - \sum (c_j/2))$ otherwise.

2.2.2 Constrained egalitarian rule.

This rule is inspired by the solution of the uniform rule (Sprumont 1991). As in Pinile´s the half-claims have a central role when the amount available is less or equal to the half of the sum of the claims and otherwise it makes minimal changes in the formula for the uniform rule to guarantee that are ordered as claims are.

Definition 11: For all $(c, E) \in \beta^N$ and all $i \in N$, $CE_i(c, E) = \min\{c_i/2, \lambda\}$ if $E <= \sum(c_j/2)$ and $CE_i(c,E) = \max\{c_i/2, \min\{c_i, \lambda\}\}$ otherwise, where in each case, λ is chosen so that $\sum CE_i(c, E) = E$.

2.2.3 Constrained Equal losses rule.

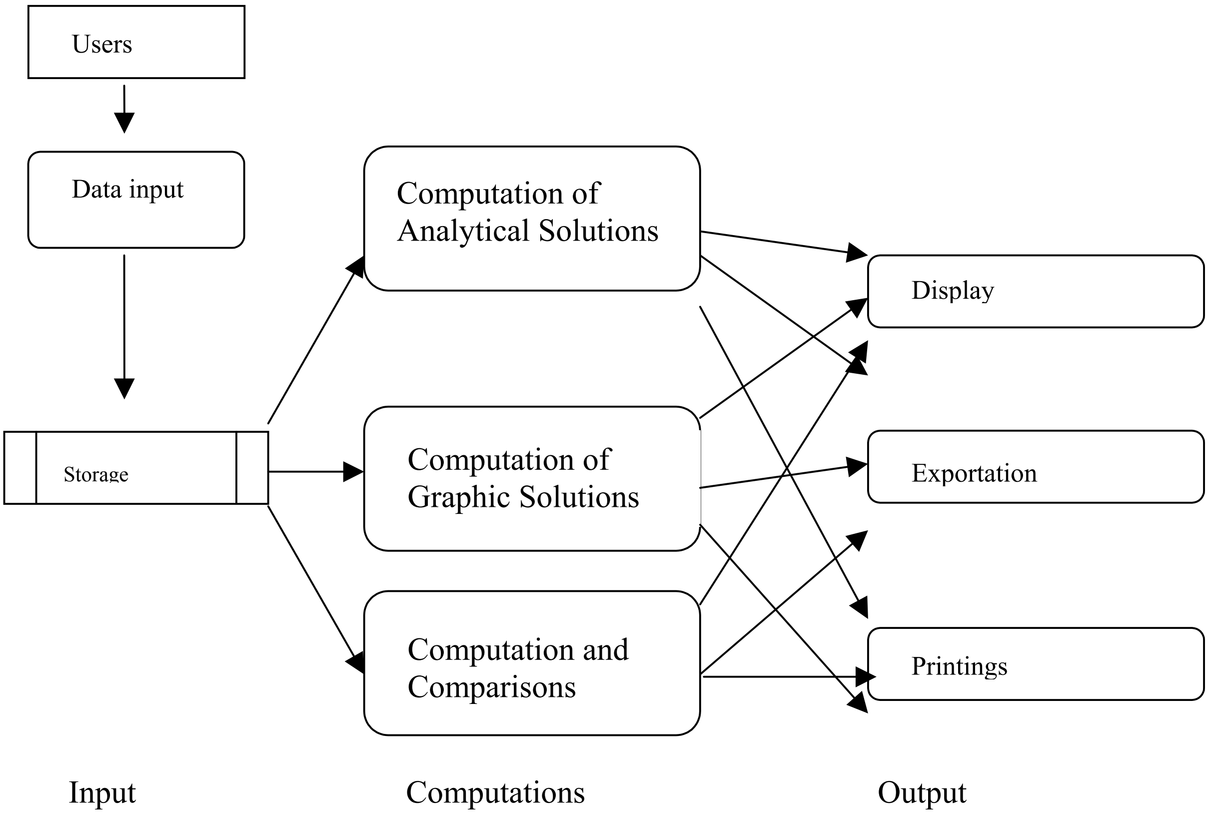
This is an alternative of the last rule, that focuses on the losses claimants incurs instead of in what they receives.

Definition 12: For all $(c, E) \in \beta^N$ and all $i \in N$, $CEL_i(c, E) = \max\{0, c_i - \lambda\}$ where λ is chosen so that $\sum \max\{0, c_i - \lambda\} = E$.

3.- The Computer System.

The computer system for bankruptcy problems is a software that computes the way of sharing a value among n agents according to the rules described above. The system is developed for a Windows environment, using Delphi language. It is multiuser, requires 7 megabytes of free disk space for its installation on a PC with at least 64 Mb of RAM There is available a CD for its installation and the user manual consisting of 35 pages.

Figure 1: Data and Process diagram



[6] Piniles, H.M. (1861)

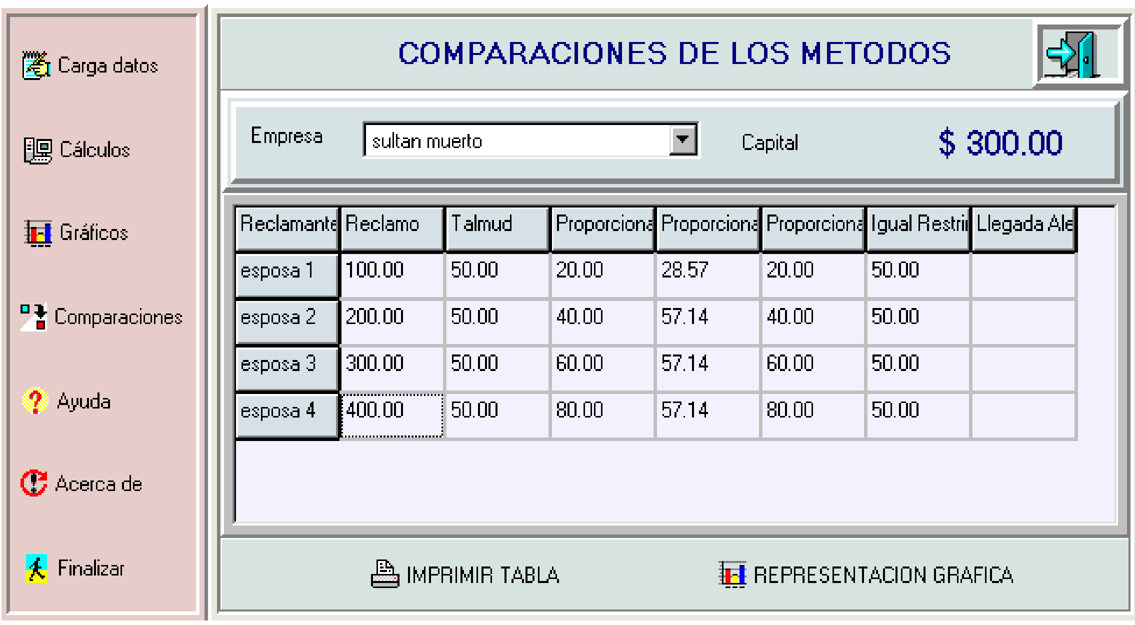
In figure 1, we show the data flow and the process diagram of the system developed. The system performs allocations to the agents regarding to different bankruptcy rules whose algorithms are described in this paper. The agents may be "claimants" (creditors) in the case of a firm that goes bankruptcy.

Now, we show some screens of the implemented computer system:

FIGURE 2 : CALCULATIONS SCREEN

After chosen " Cálculos" at the left panel, the user can choose the firm (enterprise) at the ListBox of the center panel and he will see the claimants and their claims under the worth of the estate. Then, the user can choose the rule to compute the allocations at the TabControl of the right panel.

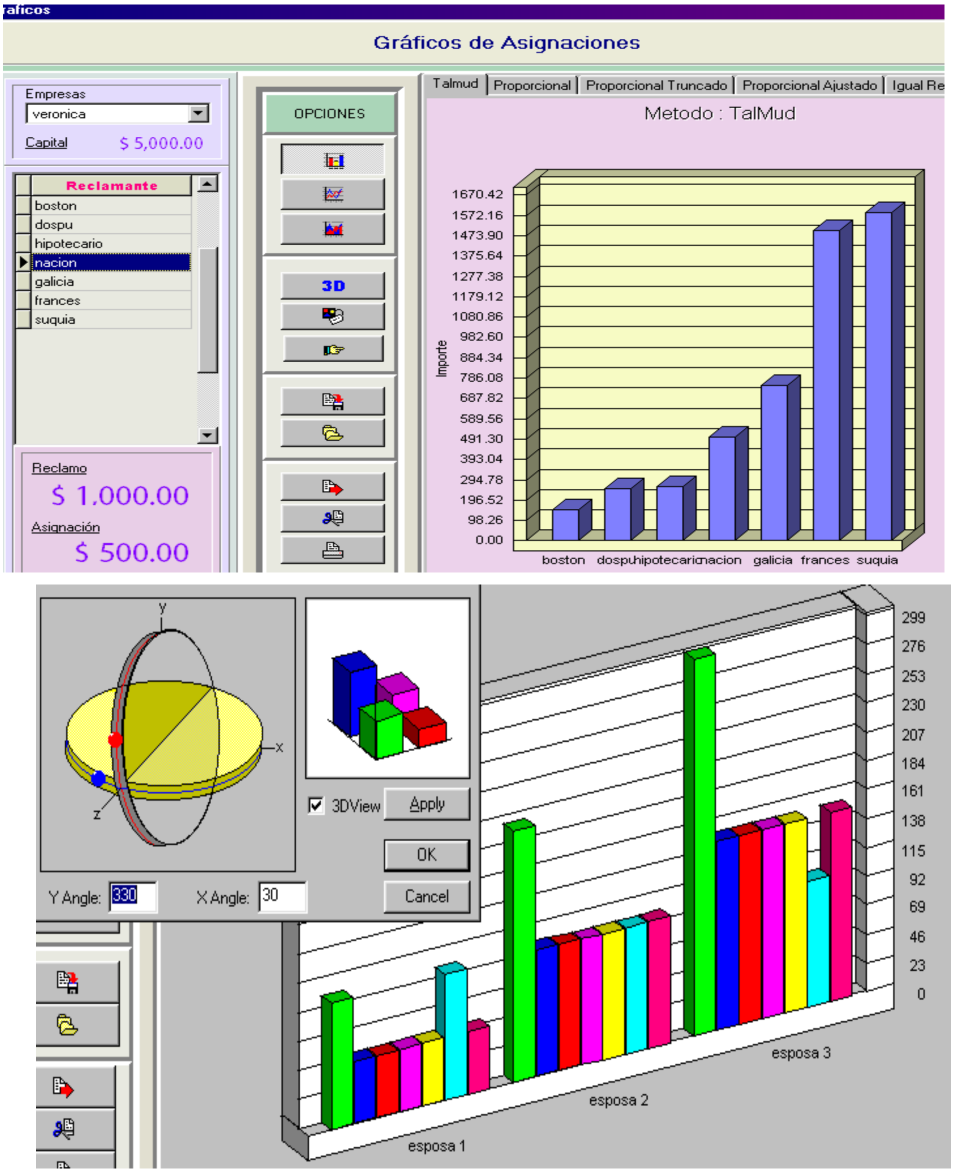
FIGURE 3: COMPARATIVE RESULTS TABLE SCREEN



After chosen “Comparaciones” at the left panel, the user can choose the firm (enterprise) at the ListBox of the right panel and he will see the worth of the estate on the right and a table with the claimants, their claims and the awards assigned by all the rules, below. Under that table, he can select to print that table or to see the graphic representation of the values assigned.

If the user chooses “Gráficos” at the left panel of figure 3, what he will obtain at the right panel is what is shown in figure 4.

FIGURE 4: ASIGNATION GRAPHIC CORRESPONDING TO THE SELECTED RULE



The second panel in figure 4 is for changing the graphic characteristics such as graphic type, 3-D, 2-D, showing all claimants, showing a group of claimants, printing the graphic, exporting graphics and values, saving graphics and values, angle selector as shown in figure 5, etc.

4.- Applications.

Using the system described above we will show different solutions for some numerical problems. The results can be exported to other applications.

Now we will consider an hypothetical case of a firm having several claimants who claim more than the worth of the estate.

Let´s suppose that a firm "Veronica 7" goes bankrupt. The worth of the state is $5.000 and the claimants are seven bank named in the first column of. Figure 6, who claim all together a total amount of $10.325. The claims are shown in the second column. The allocations computed by the system are shown in the remaining columns.

FIGURE 6: ANALYTICAL RESULTS

Claimants		Rules					
Banks	Claims	Talmud	Proportional	Truncated Proportional	Adjusted Proportional	Equal Awards	Random Arrival
Boston	300	150	145,28	145,28	145,28	300	142,86
Banex	500	250	242,13	242,13	242,13	500	242,44
Suquia	525	262,5	254,24	254,24	254,24	525	254,11
Nacion	1000	500	484,26	484,26	484,26	918,75	486,61
Galicia	1500	750	726,39	726,39	726,39	918,75	720,77
Frances	3000	1500	1452,8	1452,78	1452,78	918,75	1459,52
Rio	3500	1587,5	1694,9	1694,92	1694,92	918,75	1693,69

Figure 7 shows a comparative graphic of the analytical results shown in figure 6.

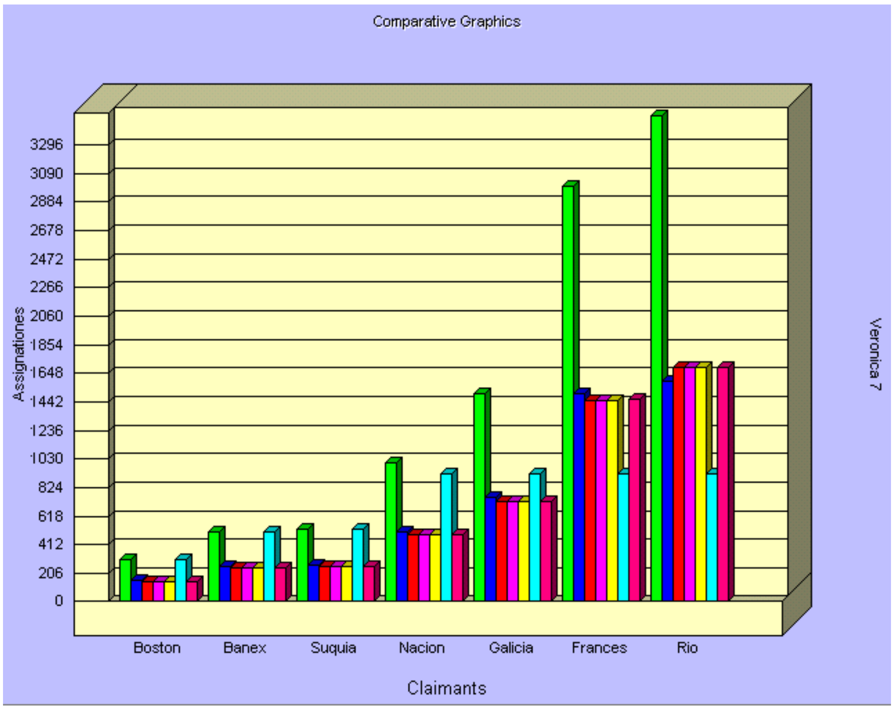


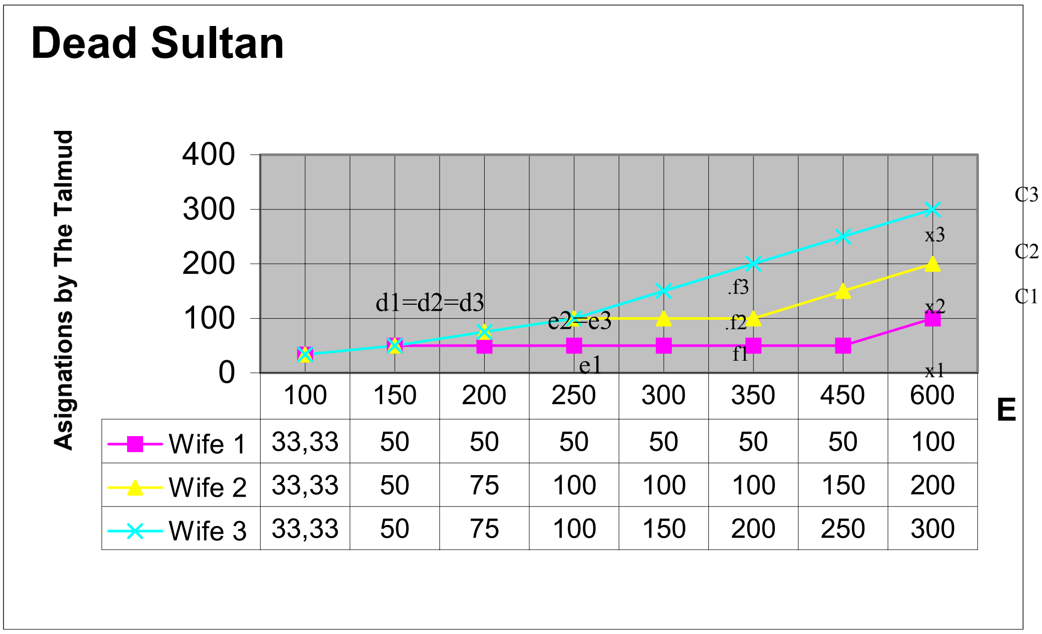
FIGURE 7: GRAPHICAL RESULTS

In the graphic shown above we can see the assignment to each claimant, computed by the program for each rule. Figures 6 and 7 were directly generated by the computer system.

Having both, the analytical and graphic comparison, the final decision can be taken considering the different allocations.

Now we will use the Computer System to analyze a historical problem discussed in the Talmud, named The Estate Division Problem: A man has three wives (the Sultan) whose marriage contracts specify that in case of his death they should receive \$100, \$200 and \$300 respectively. The Sultan dies and his estate is found to be worth only \$100. How should the amount be divided among his wives? See the figure 8.

FIGURE 8: ESTATE DIVISION PROBLEM



The claims are (c1,c2,c3) = (100, 200, 300) , if the estate is worth \$100: E = 100, (d1, d2, d3) = (33.33, 33.33, 33.33) if it is worth E = 200, The Talmud recommends (e1, e2, e3) = (50, 75, 75) and if it is \$300, E = 300 it recommends (f1, f2, f3) = (50, 100, 150).

Now we will see the results given by the System for the Estate Division Problems under different worth of Estate, using others bankruptcy rules.

FIGURE 9: AWARDS FOR THE ESTATE DIVISIÓN PROBLEM USING P

Worth of the estate		100	150	200	250	300	350	450	600
1º Wife ´s claim	100	16.67	25	33.33	41.67	50	58.33	75	100
2º Wife ´s claim	200	33.33	50	66.67	83.33	100	116.67	150	200
3º Wife ´s claim	300	50.00	75	100.00	125.00	150	175.00	225	300

FIGURE 10: AWARDS FOR THE ESTATE DIVISIÓN PROBLEM USING P^T

Worth of the estate		100	150	200	250	300	350	450	600
1º Wife ´s claim	100	33.33	37.50	40.00	45.45	50	58.33	75	100
2º Wife ´s claim	200	33.33	56.25	80.00	90.91	100	116.67	150	200
3º Wife ´s claim	300	33.33	56.25	80.00	113.64	150	175.00	225	300

FIGURE 11: AWARDS FOR THE ESTATE DIVISIÓN PROBLEM USING PA

Worth of the estate		100	150	200	250	300	350	450	600
1º Wife ´s claim	100	16.67	25	33.33	41.67	50	54.55	62.50	100
2º Wife ´s claim	200	33.33	50	66.67	83.33	100	109.09	143.75	200
3º Wife ´s claim	300	50.00	75	100.00	125.00	150	186.36	243.75	300

FIGURE 12: AWARDS FOR THE ESTATE DIVISIÓN PROBLEM USING CEA

Worth of the estate		100	150	200	250	300	350	450	600
1º Wife ´s claim	100	33.33	50	66.67	83.33	100	100	100	100
2º Wife ´s claim	200	33.33	50	66.67	83.33	100	125	175	200
3º Wife ´s claim	300	33.33	50	66.67	83.33	100	125	175	300

FIGURE 13: AWARDS FOR THE ESTATE DIVISIÓN PROBLEM USING RA

Worth of the estate		100	150	200	250	300	350	450	600
1º Wife ´s claim	100	33.33	33.33	33.33	41.67	50	58.33	66.67	100
2º Wife ´s claim	200	33.33	58.33	83.33	91.67	100	108.33	141.67	200
3º Wife ´s claim	300	33.33	58.33	83.33	116.67	150	183.33	241.67	300

FIGURE 14: AWARDS FOR THE ESTATE DIVISIÓN PROBLEM USING PIN

Worth of the estate		100	150	200	250	300	350	450	600
1º Wife ´s claim	100	33.33	50	50	50	50	66.67	100	100
2º Wife ´s claim	200	33.33	50	75	100	100	116.67	150	200
3º Wife ´s claim	300	33.33	50	75	100	150	166.67	200	300

FIGURE 15: AWARDS FOR THE ESTATE DIVISIÓN PROBLEM USING CE

Worth of the estate		100	150	200	250	300	350	450	600
1º Wife ´s claim	100	33.33	50	50	50	50	100	100	100
2º Wife ´s claim	200	33.33	50	75	100	100	125	175	200
3º Wife ´s claim	300	33.33	50	75	100	150	125	175	300

FIGURE 16: AWARDS FOR THE ESTATE DIVISIÓN PROBLEM USING CEL

Worth of the estate		100	150	200	250	300	350	450	600
1º Wife ´s claim	100	0	0	0	0	0	16,67	50	100
2º Wife ´s claim	200	0	25	50	75	100	116,67	150	200
3º Wife ´s claim	300	100	125	150	175	200	216,67	250	300

5.- Conclusions

The Computer system provides a tool for obtaining both analytical and graphic solutions to bankruptcy problems. We implemented the corresponding algorithms for each rule.

The number n of firms is only limited by the capacity of the computer disk storage. The number of claimants is also arbitrary, however , for a proper graphic representation of all the solutions we suggest to keep it under 50 agents. In case of going above it, the graphical representation should be chosen by parts.

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